D–2190

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define relatively prime integers. Give an example.

2. Define a subgroup of a group G. Give an example.

3. Define the centre of a group G.

4. Define an internal direct product of groups.

5. Show that the group of order 21 is not simple.

6. Define a ring homomorphism.

7. If D is an integral domain with finite characteristic, prove that the characteristic of D is a prime.

8. Define a left-ideal of R.

9. If $p(x)=1+x-x^2$ and $q(x)=2+x^2+x^3$, then find $p(x)q(x)$.

10. Prove that an Euclidean ring posses an unit element.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a is relatively prime to b but $a \mid bc$, then prove that $a \mid c$.

Or

(b) State and prove the Cauchy's theorem for abelian group.

12. (a) Show that the subgroup N of G is a normal subgroup of G if and only if every left-coset N in G. is a right coset N in G.

Or

(b) State and prove Lagrange's theorem.

13. (a) Prove that a ring homomorphism $\phi : R \to R'$ is one-to-one if and only if the Kernal of ϕ is zero submodule.

Or

(b) Prove that a finite integral domain is a field.

14. (a) If $[a, b] = [a', b']$ and $[c, d] = [c', d']$ then prove that $[a, b][c, d] = [a', b'][c' d']$.

Or

(b) Let R, R' be rings and ϕ a homomorphism of R onto R' with Kernal V. Prove that R' is isomorphic of R/V.

15. (a) State and prove the Einstein criterion theorem.

Or

(b) If R is an Euclidean domain, prove that any two elements a and b in R have greatest common divisor.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Cayley's theorem.

17. If p is a prime number and $\frac{p^{\alpha}}{D(G)}$, then prove that G has a subgroup of order p^{α}.

18. Define an integral domain and a Euclidean ring. Prove that any field is an integral domain.

19. If R is a unique factorization domain, then prove that $R[x]$ is also an unique factorization domain.

20. (a) State and prove Gauss lemma.

(b) State and prove the division algorithm.

D–2191

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded set. Give an example.

2. Prove that $z\,\overline{z}$ is real and positive, except when $z = 0$, where z is a complex number.

3. Define convex set.

4. Prove that in a metric space convergence sequence is bounded.

5. Define Cauchy sequence.

6. Prove that $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$.

7. Define discontinuity of the first kind at x.

8. Define closed balls in R^n.

9. State the mean value theorem.

10. Define continuously differentiable function.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $x \in R, y \in R$ and $x < y$, then prove that there exists a $P \in Q$ such that $x < p < y$.

Or

(b) Let z and w be complex numbers. Prove that $|z + w| \leq |z| + |w|$.

12. (a) Prove that every infinite subset of a countable set is countable.

Or

(b) Prove that closed subsets of compact sets are compact.

13. (a) If X is a compact metric space and if $\{p_n\}$ is a cauchy sequence in X, then prove that $\{p_n\}$ converges to some point of X.

Or

(b) Prove that, a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

14. (a) Let f be a continuous real valued function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $z(f)$ is closed.

Or

(b) Let f be monotonic on (a, b). Prove that the set of points of (a, b) at which f is discontinuous is at most countable.

15. (a) State and prove intermediate value theorem.

Or

(b) State and prove Rolle's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that every k-cell is compact.

17. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

18. Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

19. State an prove Heine-Borel covering theorem.

20. State and prove Taylor's theorem.

D–2192

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find all real valued solutions of the equation $y'' - y = 0$.

2. Find two linearly independent solutions of $y'' + \frac{1}{4x^2} y = 0$ $(x > 0)$.

3. State the existence theorem for initial value problem.

4. Compute the Wronskian of $\phi_1(x) = x^2$, $\phi_2(x) = 5x^2$.

5. Define indicial polynomial.

6. Find the singular points of the equation $x^2 y'' + (x + x^2) y' - y = 0$.

7. Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = y^2$, $y(0) = 0$.

8. Find the integrating factor of the equation

$\cos x \cos y\, dx - 2 \sin x \sin y\, dy = 0$.

9. State the local existence theorem for the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ on z.

10. When you say that a solution exists non-locally?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $L(y) = 0$ be an n^{th} order differential equation on an interval I. Show that there exist linearly independent solution of $L(y) = 0$ on I.

Or

(b) Find all the solutions of the equation :

$y'' - 4y' + 5y = 3e^{-x} + 2x^2$.

12. (a) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n\left[(n!)^2\right]}$.

13. (a) Prove that, between any two positive zeros of J_0 there is a zero of J_1.

Or

(b) Show that -1 and $+1$ are the regular singular points of the Legendre equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

14. (a) Find the integrating factor of $(2xy^3+2)dx+3xy^2dy=0$ and solve it.

Or

(b) Prove that a function ϕ is a solution of the initial value problem $y'=f(x, y)$, $y(x_0)=y_0$ on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t, y)dt$ on z.

15. (a) Let f be a real valued continuous function on the strip $S:|x|\leq\infty, |y|<\infty$, $\alpha<0$, and suppose f satisfies a lipschitz condition on S. Show that the solution of the initial value problem $y'+\lambda^2 y=f(x, y), y(0)=0$, $y'(0)=1, \lambda>0$ is unique.

Or

(b) Consider $y_1' = 3y_1 + xy_3,\ y_2' = y_2 + x^3 y_3,\ y_3' = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the two linearly independent power series solution of the equation $y''-xy'+y=0$.

17. With usual notations, prove that $J_0(x)=\sum_{m=0}^{\infty}\frac{(-1)^m}{(m!)^2}(x/2)^{2m}$.

18. If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, and $\phi_1(x) \neq 0$ on I, prove that a second solution $\phi_2(x) = \phi_2(x) \int_{x_0}^{x} \frac{1}{[\phi(s)]^2} \exp\left[-\int_{x_0}^{x} a_1(t)dt \right] ds$. Also prove that the functions ϕ_1, ϕ_2 form a basis for the solutions of $L(y) = 0$ on I.

19. Derive the indicial polynomial for the Euler equation.

20. Let f be a real valued continuous function on the strip $S : |x - x_0| \leq \alpha,\ |y| < \infty \ (a > 0)$ and suppose that f satisfies on S a Lipschitz condition with constant $k > 0$. Prove that the successive approximation $\{\phi_k\}$ for the problem $y' = f(x, y),\ y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leq a$ and converge these to a solution ϕ of $y' = f(x, y),\ y(x_0) = y_0$.

D–2193

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define onto function. Give an example.

2. Define uncountable set. Give an example.

3. State the well ordering theorem.

4. Define the discrete topology. Give an example.

5. What is meant by the product topology?

6. Define connected and path connected sets.

7. Define compact space.

8. State the tube lemma.

9. Define first countability axiom.

10. Define normal space.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a countable union of countable sets is countable.

Or

(b) Prove that every non-empty finite ordered set has the order type of a section $\{1, 2, \ldots, n\}$ of $\mathbf{Z}_+$, so it is necessarily welt ordered.

12. (a) Let X be a topological space; let A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

Or

(b) Show that lower limit topology τ' on R is strictly finer than the standard topology τ.

13. (a) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

Or

(b) Prove that every finite point set in a Hausdorff space X is closed.

14. (a) Prove that the union of a collection of connected sets that have a point in common is connected.

Or

(b) State and prove maximum and minimum value theorem.

15. (a) Show that if X is Lindelöf and Y is compact, then $X \times Y$ is Lindelöf.

Or

(b) Prove that every compact Hausdorff space is normal.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If A is finite, then prove that there is no bijection of A with a proper subset of itself.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric δ are the same as the product topology on $\mathbb{R}^n$.

18. State and prove the sequence lemma.

19. If L is a linear continuum in the order topology, then prove that L is connected and so is every interval and ray in L.

20. State and prove the Urysohn Lemma.

D–2194

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. In a vector space, show that $\infty(v-w)=\infty v-\infty w$.

2. Define subspace of a vector space. Give an example.

3. Define inner product space.

4. Define an orthonormal set.

5. Define an algebraic number.

6. For any $f(x), g(x)\in F[x]$ and any $\infty\in F$, prove that $(f(x)+g(x))'=f'(x)+g'(x)$.

7. Express the polynomial $x_1^3+x_2^3+x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

8. Define the range of T.

9. Define self adjoint and skew-Hermitian.

10. Define characteristic root of T.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the intersection of two subspaces of V is a subspace of V.

Or

(b) Prove that $\|\alpha u\| = |\alpha| \|u\|$.

12. (a) State and prove Schwarz inequality.

Or

(b) If $\{V_i\}$ is an orthonormal set, then prove that the vectors in $\{V_i\}$ are linearly independent.

13. (a) State and prove remainder theorem.

Or

(b) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

14. (a) Prove that K is a normal extension of F if and only if K is the spilitting field of some polynomial over F.

Or

(b) Prove that $G(K, F)$ is a subgroup of the group of all automorphism of K.

15. (a) If $< vT, vT > = < v,v >$ for all $v \in V$, then prove that T is unitary.

Or

(b) If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $v_1, v_2, \ldots, v_n$ is a basis of V over F and if $w_1, w_2, \ldots, w_m$ in V are linearly independent over F, then prove that $m \leq n$.

17. Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F, moreover, prove that $[L:F]=[L:K][K:F]$.

19. State and prove the fundamental theorem of Galoi's theory.

20. For every prime number p and every positive integer m there is a unique field having p^m elements.

D–2195

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Riemann – Stieltjes integral.

2. Define rectifiable curve.

3. Give an example of a sequence which is not equicontinuous.

4. Show that e^x is continuous and differentiable for all x.

5. Define equicontinuous families of functions with an example.

6. Prove that the function E is periodic, with period 2π i.

7. If $m * E = 0$, then prove that E is measurable.

8. Define outer measure.

9. Define Lebesgue integral.

10. With usual notations, prove that $|f| = f^+ + f^-$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If p^* is a refinement of p, then prove that $L(p,\ f,\ \alpha) \leq L(p^*, f, d)$.

Or

(b) State and prove integration by parts theorem.

12. (a) State and prove the Fundamentals theorem of calculus.

Or

(b) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval, but does not converges absolutely for any value of x.

13. (a) Suppose Σc_n converges. Put

$f(x) = \sum_{n=0}^{\infty} c_n x^n,\ (-1 < x < 1)$. Prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

Or

(b) Define the gamma function $\Gamma(x)$ and prove that $\Gamma(x+1) = x\Gamma(x), \Gamma(1) = 1$ and $\log \Gamma(x)$ is convex.

14. (a) Show that the interval (a, ∞) is measurable.

Or

(b) Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*\left(\bigcup A_n\right) \leq \sum m^* A_n$.

15. (a) State and prove Fatou's lemma.

Or

(b) Suppose $f = f_1 + f_2$, where $f_1 \in L(\mu)$ on $E(i = 1, 2)$. Prove that $f \in L(\mu)$ on E, and

$$\int_E f \, d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu .$$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. State and prove the Stone-Weierstrass theorem.

18. State and prove Parseval's theorem.

19. State and prove Lusin's theorem.

20. State and prove Lebesgue's monotone convergence theorem.

D–2196

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define the one point compactification. Give an example.

2. What is meant by completely regular space?

3. State countable intersection property.

4. Define an open refinement. Give an example.

5. Define the stone – cech compactification.

6. When will you say cauchy sequence is complete?

7. Define an equicontinuous function.

8. State totally bounded in a metric space.

9. Define Baire space with an example.

10. Define an M-cube.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define locally compact with an example. Show that the rationals Q are not locally compact?

Or

(b) Prove that a subspace of a completely regular space is completely regular.

12. (a) Under what conditions does a metrizable space have a metrizable compactification.

Or

(b) Prove that every paracompact Hausdorff space X is normal.

13. (a) Define F_σ – set. Also prove that W is an F_σ – set in X if and only if $X - W$ is a G_δ set.

Or

(b) Prove that the Euclidean space R^K is complete in either of its usual metrics, the Euclidean metric d or the square metric δ.

14. (a) Show that in the compact open topology, $\zeta(X, Y)$ is Hausdorff if Y is Hausdorff and regular if Y is regular.

Or

(b) Prove that any open subset Y of a Baire space X is itself a Baire space.

15. (a) If Y is a closed subset of X, and if X has finite dimension then so does Y; and $\dim Y \leq \dim X$: Prove.

Or

(b) Prove that every compact subset of $\mathbb{R}^N$ has topological dimension at most N.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that an arbitrary product of compact space is compact in the product topology.

17. State and prove Tietze extension theorem.

18. State and prove the Smirnov metrization theorem (necessary part).

19. State and prove Baire category theorem.

20. Let $h:[0,1] \to R$ be a continuous function. Given $\in > 0$, prove that there is a function $g:[0,1] \to R$ with $|h(x) - g(x)| < \in$ for all x, such that g is continuous and nowhere – differentiable.

D–2197

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define exact differential equation.

2. Show that the equation $\left(x^2 z - y^3\right)dx + 3xy^2 dy + x^3 dz = 0$ is integrable.

3. Define oblique trajectory.

4. Solve : $2p + 3q = 1$.

5. What is the general solution of

   $p_1 p_1 + p_2 p_2 + ... + p_n p_n = R$?

6. Write the subsidiary equations for the equation $\dfrac{y^2 z}{x} + z\,xy = y^2$.

7. Show that the differential equations $\frac{\partial z}{\partial x} = 5x - 7y$ and $\frac{\partial z}{\partial y} = 6x + 8y$ are not compatible.

8. Solve : $\left(D^2 - 5DD' + 4D'^2\right)z = 0$.

9. Solve $\frac{\partial^2 z}{\partial x^2} = 6x$.

10. Write down the interior Dirichlet boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $\frac{dx}{xz\left(z^2 + xy\right)} = \frac{dy}{-yz\left(z^2 + xy\right)} = \frac{dz}{x^4}$.

Or

(b) Solve : $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{bx - ay}$.

12. (a) Show that $\left(2x + y^2 + 2xz\right)dx + 2xy\,dy + x^2 dz = 0$ is integrable.

Or

(b) Form a partial differential equation by eliminating the arbitrary function f from the equation $x + y + z = f\left(x^2 + y^2 + z^2\right)$.

13. (a) Find the integral surface of the linear partial differential equation

$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0,\ z=1$.

Or

(b) Find the family of surfaces orthogonal to family of surfaces given by the differential equation $(y+z)p+(z+x)q = x+y$.

14. (a) Solve : $p_1 + p_2 + p_3 = 4z$.

Or

(b) Find the complete integral of $py + qx + pq = 0$.

15. (a) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive D'Alembert's solution of one-dimensional wave equation.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is a parameter.

17. Solve : $\frac{dx}{x+y-xy^2} = \frac{dy}{x^2y-x-y} = \frac{dz}{z(y^2-x^2)}$.

18. Obtain the complete integral of $p^2 + q^2 - 2px - 2qy + 1 = 0$.

19. Solve : $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$.

20. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = y_0 \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement at time t.

D–2198

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Osculating plane.

2. What is meant by Torsion?

3. Define evolute.

4. Define are length.

5. Define archor ring.

6. What is meant by the binormal line?

7. Define a hyperbolic point.

8. Write short notes on the geodesic curvature.

9. Define principal curvatures.

10. Define the osculating developable of the curve.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Serret-Frenet formula.

Or

(b) With the usual notations prove that $[\vec{r}', \vec{r}'', \vec{r}'''] = k^2 r$.

12. (a) If a curve lies on a sphere, show that δ and σ are related by $\frac{d}{ds}(\sigma \delta') + \frac{\delta}{\sigma} = 0$.

Or

(b) Show that the involutes of a circular helix are plane curves.

13. (a) Calculate the fundamental coefficients E, F, G and H for the paraboloid $\vec{r} = (u, v, u^2 - v^2)$.

Or

(b) Discuss the geodesic parallels.

14. (a) Find the Gaussian curvature at (u, v) of the achor ring.

Or

(b) Enumerate the geodesic polar form. Also find the circumference of a geodesic circle of small radius r.

15. (a) State and prove Euler's theorem.

Or

(b) Show that the characteristics point of the plane u is determined by the equations $\vec{r} \cdot \vec{a} = p$, $\vec{r} \cdot \vec{a} = \dot{p}, \vec{r} \cdot \ddot{\vec{a}} = \ddot{p}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that the intrinsic equations of the curve given by $x = a e^u \cos u$, $y = a e^u \sin u$, $z = b e^u$ are $k = \frac{\sqrt{2}\, a}{\left(2a^2 + b^2\right)^{1/2}} \cdot \frac{1}{s}$, $\tau = \frac{b}{\left(2a^2 + b^2\right)^{1/2}} \cdot \frac{1}{s}$.

17. Derive the differential equations for a geodesic using the normal property.

18. Find the surface of revolution which is isometric with a region of the right helicoids.

19. State and prove the Gauss-Bonnet theorem.

20. Show that a necessary sufficient condition for a surface to be developable is that its Gaussian curvature shall be zero.

D–2199

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2023.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by connected network?

2. Define the total float and the free float.

3. Write two features of O.R.

4. Define critical activity.

5. Define critical path.

6. What is minimax strategy?

7. What is interval of uncertainty?

8. Explain dichotomous search method.

9. Define general constrained non-linear programming problem.

10. What is meant by steepest method?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain three jug puzzle with an illustration.

Or

(b) Construct the network diagram comprising activities B, C,...,Q and N such that following constraints are satisfied.

B < E, F; C < G, L; E, G < H; L < H < I; L < M; H < N, H < J; I, J < P; P < Q. The notation $x < y$. Means that the activity x must be finished before y can begin.

12. (a) Solve :

Minimize : $z = x_1 + x_2 + 2x_3$

Subject to the constraints :

$$\begin{aligned} x_1 + x_2 + x_3 &\le 9 \\ 2x_1 - 3x_2 + 3x_3 &= 1 \\ -3x_1 + 6x_2 - 4x_3 &= 3; \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

Or

(b) Solve :

Maximize $z = 5x_1 + 2x_2$

Subject to the constraints :

$$6x_1 + x_2 \geq 6$$
$$4x_1 + 3x_2 \geq 12$$
$$x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0.$$

13. (a) Solve the following game and determine the value of the game :

Player B

$$\text{Player A} \begin{pmatrix} 2 & 5 \\ 7 & 3 \end{pmatrix}$$

Or

(b) Is the following two-person zero-sum game stable? (The payoff is for player A). Solve the game problem :

Player B

$$\text{Player A} \begin{pmatrix} 8 & 6 & 2 & 8 \\ 8 & 9 & 4 & 5 \\ 7 & 5 & 3 & 5 \end{pmatrix}$$

14. (a) Find the minimax and maximin for the following data :

$$\begin{pmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{pmatrix}$$

Or

(b) Solve the LPP by Lagrangion method :

Maximize : $f(x) = 5x_1 + 3x_2$

Subject to :

$g_1(x) = x_1 + 2x_2 + x_3 - 6 = 0$
$g_2(x) = 3x_1 + x_2 + x_4 - 9 = 0$
$x_1, x_2, x_3, x_4 \geq 0.$

15. (a) Prove that a necessary condition for X_0 to be an extreme point of $f(X)$ is that $\nabla f(x_0) = 0$.

Or

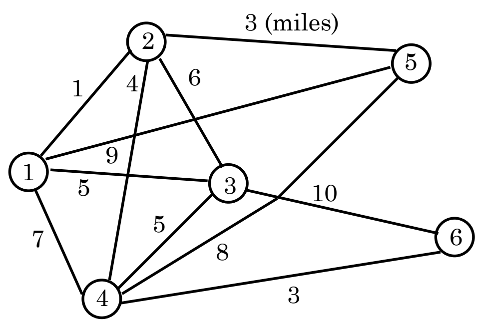
(b) Explain a quadratic programming model problem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the minimal spanning tree of the following network under the independent conditions :

Nodes 2 and 6 are linked by a 4-mile cable.



D–2199

17. The footings of a building can be completed in four consecutive sections :

The activities for each section include digging, placing steel, and pouring concrete. The digging of one section cannot start until the proceeding one is completed. The same restriction applies to pouring concrete. Develop a network for the project.

18. Solve the problem by the revised simplex method :

Minimize : $z = 2x_1 + x_2$

Subject to :

$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0.$

19. Apply the Newton-Raphason method to solve :

$f(x) = 2x_1^2 + x_2^2 + x_3^2 + 6(x_1 + x_2 + x_3) + 2x_1 x_2 x_3 .$

Examine the functions for extreme points.

20. Write the Kuhn-Tucker necessary conditions for the problem :

Maximize $f(x) = x_1^3 - x_2^2 + x_1 x_3^2$

Subject to :

$x_1 + x_2^2 + x_3 = 5$

$5x_1^2 - x_2^2 - x_3 \geq 0$

$x_1, x_2, x_3 \geq 0.$

D–2200

Sub. Code
31133

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is meant by divisibility? Give an example.
2. Define the Mobius function $\mu(n)$.
3. Prove that $\phi(m,n) = \phi(m) \,.\, \phi(n)$ if $(m, n) = 1$.
4. Define Liouville's function $\lambda(n)$.
5. State the Little Fermat theorem.
6. Prove that $[x+n] = [x] + n$.
7. What are the solutions of the congruence $x^2 \equiv 1 (\bmod 8)$?
8. Write down the Legendre's symbol (n/p).
9. What are the quadratic residues and non-residues mod 13?
10. Define the Jacobi symbol.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) State and prove Euclidean algorithm.

12. (a) Prove that $d(n)$ is odd if and only if n is a square.

Or

(b) Let f be multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

13. (a) State and prove Euler's summation formula.

Or

(b) Prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$, if $x \geq 1$.

14. (a) Prove that $5n^3 + 7n^2 \equiv 0 \pmod{12}$ for all integer n.

Or

(b) State and prove Chinese remainder theorem.

15. (a) Determine whether 219 is a quadratic residue or non-residue mod 383.

Or

(b) State and prove reciprocity law for Jacobi symbol.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

17. State the Mobius inversion formula. Also derive the Selberg identify.

18. Derive Dirichets' asymptotic formula.

19. State and prove Wilson's theorem.

20. Prove that the diaphantine equation $y^2 = x^3 + K$ has no solution if K has the form $K = (4n-1)^2 - 4m^2$ where m and n are integers such that no prime $p \equiv -1 \pmod 4$ divides m.

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D–2201

Sub. Code
31134

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Markov chain. Give an example.

2. Define transition probability.

3. What is meant by class property?

4. Write the forward diffusion equation of the Wiener process.

5. Define sample paths.

6. What is Ornstein – Uhlenbeck process?

7. When does a Galton Watson process is a Markov chain?

8. Define a Markov branching process.

9. Explain inter arrival time.

10. What is meant by birth and death process?

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive the mean recurrence time for the state j as $\mu_{jj} = \sum_{n=1}^{\infty} n f_{jj}^{(n)}$.

Or

(b) State and explain the postulates for Poisson process.

12. (a) If $\{N(t)\}$ is a Poisson process and $s < t$ then prove that $P_r\{N(s) = K \mid N(t) = n\} = \binom{n}{k} (s/t)^K (1-(s/t))^{K-1}$.

Or

(b) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma X(t/\sigma^2)$ is also a Wiener process. Find its covariance function.

13. (a) If $m \leq 1$, the probability of ultimate extinction is 1. If $m > 1$, the probability of ultimate extinction is the positive root less than unity of the equation $s = p(s)$ – prove.

Or

(b) Show that the p.d.f. of one of the conditional distribution of X_n, given $X_n > 0$ is $\frac{P_n(s) - P_n(0)}{1 - P_n(0)}$. Find $P_r\{X_n = r \mid X_n > 0\}$. When P_r {number of offspring = K} = $\left(\frac{1}{2}\right)^{K+1}$, $K =$ 0, 1, 2....

14. (a) If $m = E(X_1) = \sum_{K=0}^{\infty} Kp_k$ and $\sigma^2 = \text{var}(X_1)$ then prove that $E\{X_n\} = m^n$ and

$$\text{var}(X_n) = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & if\ m \neq 1 \\ n\sigma^2, & if\ m = 1 \end{cases}.$$

Or

(b) Prove that the p.g.f. $R_n(s)$ of Y_n satisfies the recurrence relation $R_n(s) = sP(R_{n-1}(s))$, $P(s)$ being the p.g.f. of the offspring distribution.

15. (a) Explain about steady state distribution.

Or

(b) Explain the model $M/M/\infty$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Chapman – Kolmogorov equation.

17. Prove that state j is persistent if and only if $\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$.

18. Prove that the generating function $F(t, s) = \sum_{K=0}^{\infty} P_r\{X(t) = k\} s^K$ of an age-dependent branching process $\{X(t), t \geq 0\}$; $X_0 = 1$ satisfies the integral equation $F(t,s) = [1 - G(t)] + \int_0^t P(F(t-u, s))dG(u)$.

19. Prove that $P_n(s) = P_{n-1}(P(s))$ and $P_n(s) = P(P_{n-1}(s))$ where $P_n(s) = \sum_K P_K s^K$.

20. Derive the Erlang's second formula $C(s, \lambda/\mu) = \dfrac{P_s}{1-\rho}$.

D–2202

Sub. Code
31141

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, DECEMBER 2023.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a spanning subgraph of a graph *G*.

2. Define cut vertex of a graph. Give an example.

3. Define a path a cycle.

4. Define cut vertex and bridge of a graph.

5. Define perfect matching.

6. What is meant by covering number of a graph *G?*

7. Define the chromatic number of a graph.

8. Define planar graph.

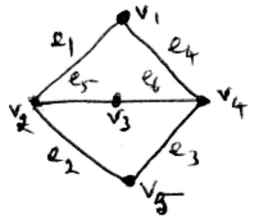
9. State the five colour theorem.

10. Define directed cycle with an example.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define incidence matrix of a graph. Also find the incidence matrix of the graph :



Or

(b) Prove that every tree has a centre consisting of either one vertex or two adjacent vertices.

12. (a) For what values of n does the complete graph K_n have perfect matching?

Or

(b) Prove that $\gamma(m,n) = \gamma(n,m)$.

13. (a) If G is uniquely n-colourable, then prove that $\delta(G) \geq n-1$.

Or

(b) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

14. (a) Prove that if e is a bridge of a connected graph G, then $G - e$ has exactly two components.

Or

(b) Show that there is not map of five regions in the plane such that every pair of regions are adjacent.

15. (a) If two diagraphs are isomorphic then prove that the corresponding vertices have the same degree pair.

Or

(b) State and prove max-flow, min-cut theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let G be a (p,q) graph. Prove that the following are equivalent.

(a) G is a tree.

(b) Every two vertices of G are joined by a unique path.

(c) G is connected and $p = q + 1$.

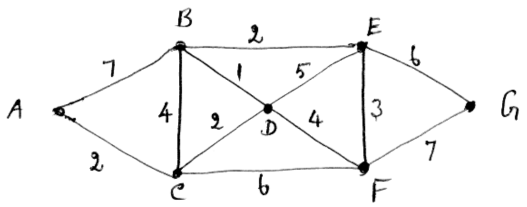
(d) G is acyclic and $p = q + 1$.

17. State and prove that Chavatal theorem.

18. State and prove Brook's theorem.

19. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively, then prove that $|V| - |E| + |F| = 2$.

20. Find the shortest distance of the vertex G from the vertex A of the following weighted graph.



D–2203

Sub. Code
31142

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, DECEMBER 2023.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded linear map.
2. Define banach Space. Give an example.
3. Define completely continuous map.
4. Define inner product space. Give an example.
5. What is meant by annihilator?
6. Define projection mapping.
7. Define Hilbert space.
8. What is meant by unitary operator?
9. State the Riez theorem.
10. State closed graph theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a normed space. If E_1 is open in X, and $E_2 \subset X$, then prove that $E_1 + E_2$ is open in X.

Or

(b) Prove that l_∞ is a complete space.

12. (a) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the rang $R(F)$ of F is finite dimensional. Prove that F is continuous if and only if the zero space $Z(F)$ of F is closed in X.

Or

(b) Prove that a Banach space cannot have denumerable basis.

13. (a) Prove that a closed subspace of a compact space is compact.

Or

(b) Let X and Y be normed space and let $F : X \to Y$ be linear. Prove that f is continuous if and only if $g o f$ is continuous for every $g \in Y'$.

14. (a) Let X be an inner product space and $f \in X'$. Let $\{v_\alpha\}$ be an orthonormal set in X and $E_f = \{u_\alpha : f(u_\alpha) \neq 0\}$. Then E_f is a countable set say $\{u_1, u_2 ....\}$. If E_f is denumerable, then prove that $f(u_n) \to 0$ as $n \to \infty$.

Or

(b) State and prove Bessel's inequality.

15. (a) Define strong convergence. Show that weak convergence does not imply strong convergence.

Or

(b) If X is a finite dimensional normed space, prove that strong convergent is equivalent to weak convergence.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that a subset A of a metric space (X,d) is sequentially compact if and only if it is compact.

17. Let X be a normed space. Prove that the following are equivalent.

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X : \|x\| \leq 1\}$ of X is compact.

(c) X is finite dimensional.

18. State and prove Hahn Banach extension theorem.

19. Let H be a Hilbert space, G be a subspace of H and g be a continuous linear functional on G. Prove that there is a unique continuous linear functional f on H such that $f \mid g = g$ and $\|f\| = \|g\|$.

20. State and prove uniform boundedness theorem.

D–2204

Sub. Code
31143

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, DECEMBER 2023.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write the sufficient condition on $\phi(x)$ for convergence in iteration methods.

2. Explain sturm sequence.

3. Write the deflated polynomial.

4. Define an eigen values of a matrix A.

5. What is the truncation error of the Lagrange quadratic interpolating polynomial?

6. What is meant by Knots?

7. What is spline fitting?

8. Write the formula for Gauss method.

9. Write the formula for second order $R-K$ method.

10. What are the merits and demerits of Taylor's method?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a root of $\cos x - x^2 - x = 0$ to five decimal places.

Or

(b) Use synthetic division and perform two iterations of the Birge – Vieta method to find the smallest positive root of $P_3(x) = 2x^3 - 5x + 1 = 0$.

12. (a) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ using the maximum absolute row sum norm.

Or

(b) Prove that no eigen value of a matrix A exceeds the norm of a matrix. (ie, $\|A\| \geq \rho(A))$.

13. (a) Derive the Hermite interpolating polynomial.

Or

(b) Using the following values of $f(x)$ and $f'(x)$, estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

14. (a) Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{1/2}$ on [0, 1].

Or

(b) The following table of values is given :

x	–1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = \left(f(x_2) - f(x_0)\right)/(2h)$ and the Richardson extrapolation, find $f'(3)$.

15. (a) Discretize the initial value problem $y' = 1 + y,\ y(1) = 0$ using backward Euler method. Compute $y(1.2)$ using $h = 0.1$.

Or

(b) Use the Taylor series method of order four to solve the initial value problem $u' = t^2 + u^2$, $u(0) = 1$ for the interval (0, 0.4) using two subintervals of length 0.2.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $P_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$.

17. Using Jacobi method, find all the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

18. Construct the bivariate interpolating polynomial and hence find $f(0.5, 0.5)$ from the following function $f(x, y)$:

$y \backslash x$	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

19. Evaluate the integral $\int_0^2 e^x dx$ using Simpson's rule with $h = 1$ and 1/2. Find a bound on the error in each case. Compare with exact solution.

20. By applying fourth order $R - K$ method find $y(0.2)$ from $y' = y - x$, $y(0) = 2$ taking $h = 0.1$.

D–2205

Sub. Code
31144

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If the sample space is $\mathcal{C} = c_1 \cup c_2$ and if $P(c_1) = 0.8$ and $P(c_2) = 0.5$, then find $P(c_1 \cap c_2)$.

2. Define the distribution function of X and Y.

3. Prove that $E[(X - \mu_1)(Y - \mu_2)] = E(XY) - \mu_1\mu_2$.

4. Let the joint p.d.f. of X_1 and X_2 be $f(x_1, x_2) = \begin{cases} 12 \quad x_1 x_2 (1 - x_2), & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$. Are x_1 and x_2 are stochastically independent? Justify.

5. Determine the binomial distribution for which the mean is 4 and variance is 3.

6. Write the p.d.f. of gamma distribution.

7. Define t-distribution.

8. What do you meant by the change of variable technique?

9. Define convergence in distribution.

10. State the central limit theorem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If X has the p.d.f. $f(x) = \begin{cases} (1/2)^x, & x = 1,2,3.... \\ 0, & \text{elsewhere} \end{cases}$ find the m.g.f, mean and variance of X.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} (x+2)/18, & -2 < x < 4 \\ 0, & \text{elsewhere} \end{cases}$. Find $E(X)$, $E\left[(x+2)^3\right]$ and $E\left[6x - 2(x+2)^3\right]$.

12. (a) Find $P_r\left(0 < X_1 < \frac{1}{3}, 0 < X_2 < 1/3\right)$ if the random variables X_1 and X_2 have the joint p.d.f. $f(x) = 4x_1(1 - x_2)$, $0 < x_1 < 1$, $0 < x_2 < 1$; zero elsewhere.

Or

(b) Compute the measure of Skewness and Kurtosis of the binomial distribution $b(n, p)$.

13. (a) Find the mean and variance of Poisson distribution.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} (1/2)^x, & x = 1,2,3.... \\ 0, & \text{elsewhere} \end{cases}$.

Find the p.d.f. of $Y = X^3$.

14. (a) Let X_1, X_2 be a random sample from a distribution having the p.d.f. $f(x) = \begin{cases} e^{-x}, & 0 < x < a \\ 0, & \text{elsewhere} \end{cases}$. Show that $Z = X_1/X_2$ has a F-distribution.

Or

(b) Derive the p.d.f. of chi-square distribution.

15. (a) Let z_n be $\chi^2(n)$ and let $w_n = \frac{z_n}{n^2}$, find the limiting distribution of W_n.

Or

(b) Let $\overline{X}_n$ denote the mean of the random variable of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, determine a lower bound for the probability $P_r(-2 < x < 8)$ using Chebyshev inequality.

17. Find the moment generating function, mean and variance of the gamma distribution.

18. Derive t – distribution and derive the p.d.f. of the t-distribution.

19. Let $T = \frac{W}{\sqrt{V/\gamma}}$ where W and V are respectively normal with mean 0 and variance 1. Chi-square with γ. Show that T^2 has an F-distribution with parameters $\gamma_1 = 1$, $\gamma_2 = \gamma$.

20. State and prove the central limit theorem.

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